surface temperature boundary conditions may be seriously in error.

ACKNOWLEDGEMENTS

The work on this investigation was supported by the United States Atomic Energy Commision, Division of Reactor Development, Engineering Development Branch, and was carried out at Argonne National Laboratory, Reactor Engineering Division.

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THE EFFECTIVENESS OF A COUNTER-FLOW HEAT EXCHANGER WITH CROSS-FLOW HEADERS

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(*Received* 6 *July* 1967 *and in revised form* 6 *November* 1967)

NOMENCLATURE

A, heat-transfer surface area on either hot or cold fluid sides;
C, fluid capacity rate ($= \dot{m}c_p$);
c_p, fluid specific heat;
C_{min}/C_{max}, capacity rate ratio;
E, heat exchanger effectiveness,

$$E = \frac{C_h(t_{h,\,in} - t_{h,\,out})}{C_{min}(t_{h,\,in} - t_{c,\,in})} = \frac{C_c(t_{c,\,out} - t_{c,\,in})}{C_{min}(t_{h,\,in} - t_{c,\,in})};$$

$\dot{m}$, fluid mass flow-rate;
N_{tu}, number of transfer units, $N_{tu} = (AU/C_{min})$; Note: A and U must be based on the same side of the heat exchanger; the product (AU) is the same on both the hot and cold sides;
U, overall unit heat-transfer conductance from hot side fluid to cold side fluid, based on a unit of area A.

Subscripts
c, "cold" fluid side;
h, "hot" fluid side;
min, the side with the minimum of either C_c or C_h.

A DIFFICULTY in building a counter-flow heat exchanger using very compact plate-fin heat-transfer surfaces arises in the separation of the two fluids at the ends of the heat exchanger. Some kind of cross-flow sandwich header must be used, and the usual procedure is to extend the plates to form one or other of the two patterns shown in Fig. 1. Then the fins in the end sections are brazed in such a manner as to turn the flow and separate the fluids as in a cross-flow heat exchanger.

The result is a composite heat exchanger with a true counter-flow central core, but with cross-flow headers. One can anticipate that the performance, based on the *total* heat-transfer surface area, would be inferior to that of a

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simple counter-flow arrangement, but superior to a simple cross-flow arrangement. Frequently in gas-flow applications the flow frontal area must be so large that a substantial portion of the total heat-transfer surface area (large portion of the total N_{tu}) is in the cross-flow header section, and it is thus desirable to have a reasonably accurate solution for such a composite heat exchanger.

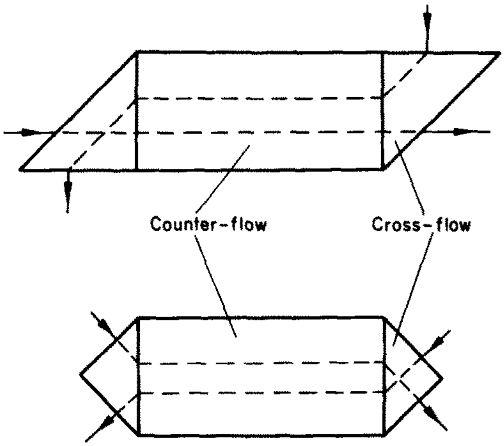

FIG. 1. Two commonly used methods to separate the fluids at the ends of a counter-flow heat exchanger using cross-flow headers.

A popular approximate procedure has been to treat the heat exchanger as a counter-flow heat exchanger in series with two (or perhaps one) cross-flow heat exchanger, but this becomes an uncertain expedient at high values of effectiveness.

The objective of this note is to present the results of a numerical solution of such a composite heat exchanger, a solution that avoids any approximations other than those inherent in the usual heat exchanger idealizations, and in the finite difference method of calculation. It is assumed that fluid properties are constant, that the fluid flow-rates are evenly distributed over the frontal area, and that the heat-transfer surface area is uniformly distributed over the counter-flow portion, and over the cross-flow positions, although the area density may be different in the two parts. It is also assumed that the unit overall conductance, U, is constant over the counter-flow portion, and over the cross-flow portions, but not necessarily the same in the two portions.

The numerical procedure was necessary only for the end sections, since the exact counter-flow algebraic solution could be applied to each of the flow tubes in the counter-flow central core. A 10 × 10 mesh was employed for the end sections, with the heat-flux evaluated for each cell using the difference of the arithmetic averages of the terminal fluid temperatures, i.e. central differences. The adequacy of this mesh and the method of calculation was established by two tests. A trial calculation with a 50 × 50 mesh introduced differences in the calculated effectiveness, E, only in the fourth significant figure. Calculations with

Table 1. E for C_{min}/C_{max} = 0·20

N_{tu}, counter-flow	N_{tu}, cross-flow 0	1	2	3	4	5	6	7
0	0	0·597	0·811	0·903	0·946	0·969	0·982	0·989
1	0·605	0·825	0·914	0·955	0·975	0·986	0·992	0·995
2	0·834	0·923	0·961	0·979	0·989	0·993	0·996	0·998
3	0·927	0·966	0·983	0·991	0·995	0·997	0·998	0·999
4	0·967	0·985	0·992	0·996	0·998	0·999	0·999	1·000
5	0·985	0·993	0·997	0·998	0·999	0·999	1·000	1·000
6	0·995	0·997	0·998	0·999	1·000	1·000	1·000	1·000
7	0·998	0·999	0·999	1·000	1·000	1·000	1·000	1·000

Table 2. E for C_{min}/C_{max} = 0·40

N_{tu}, counter-flow	N_{tu}, cross-flow 0	1	2	3	4	5	6	7
0	0	0·564	0·759	0·848	0·898	0·928	0·948	0·961
1	0·577	0·783	0·871	0·916	0·942	0·959	0·970	0·978
2	0·795	0·887	0·930	0·954	0·968	0·977	0·983	0·988
3	0·897	0·940	0·962	0·975	0·982	0·987	0·991	0·993
4	0·945	0·968	0·979	0·986	0·990	0·993	0·995	0·996
5	0·970	0·982	0·989	0·992	0·995	0·996	0·997	0·998
6	0·985	0·990	0·994	0·996	0·997	0·998	0·998	0·999
7	0·991	0·995	0·997	0·998	0·998	0·999	0·999	0·999

Table 3. E for $C_{min}/C_{max} = 0{\cdot}60$

N_{tu}, counter-flow	N_{tu}, cross-flow 0	1	2	3	4	5	6	7
0	0	0·532	0·708	0·792	0·842	0·875	0·898	0·916
1	0·552	0·739	0·821	0·867	0·896	0·917	0·931	0·943
2	0·754	0·843	0·888	0·914	0·932	0·945	0·954	0·962
3	0·854	0·902	0·928	0·944	0·955	0·963	0·969	0·974
4	0·908	0·937	0·953	0·963	0·970	0·976	0·980	0·983
5	0·940	0·959	0·969	0·976	0·980	0·984	0·986	0·988
6	0·963	0·973	0·980	0·984	0·987	0·989	0·991	0·992
7	0·975	0·982	0·987	0·989	0·991	0·993	0·994	0·995

Table 4. E for $C_{min}/C_{max} = 0{\cdot}8$

N_{tu}, counter-flow	N_{tu}, cross-flow 0	1	2	3	4	5	6	7
0	0	0·503	0·660	0·736	0·783	0·815	0·838	0·856
1	0·512	0·694	0·768	0·811	0·840	0·860	0·876	0·889
2	0·712	0·793	0·835	0·861	0·880	0·894	0·905	0·914
3	0·805	0·852	0·878	0·896	0·908	0·918	0·926	0·932
4	0·860	0·891	0·908	0·920	0·929	0·936	0·942	0·947
5	0·895	0·917	0·929	0·938	0·945	0·950	0·954	0·958
6	0·922	0·936	0·945	0·951	0·956	0·960	0·963	0·966
7	0·940	0·950	0·957	0·961	0·965	0·968	0·971	0·973

Table 5. E for $C_{min}/C_{max} = 1{\cdot}00$

N_{tu}, counter-flow	N_{tu}, cross-flow 0	1	2	3	4	5	6	7
0	0	0·476	0·615	0·682	0·723	0·752	0·773	0·790
1	0·500	0·649	0·714	0·751	0·777	0·795	0·810	0·821
2	0·667	0·739	0·775	0·798	0·815	0·828	0·838	0·846
3	0·750	0·792	0·816	0·831	0·843	0·852	0·860	0·866
4	0·800	0·828	0·844	0·855	0·863	0·870	0·876	0·880
5	0·834	0·853	0·865	0·873	0·880	0·885	0·889	0·893
6	0·856	0·872	0·881	0·887	0·892	0·897	0·900	0·903
7	0·875	0·886	0·893	0·899	0·903	0·906	0·909	0·912

the counter-flow core absent, i.e. a simple cross-flow heat exchanger, yielded results for the effectiveness that differed from the established solution [1] for this case by no more than one integer in the third significant figure.

The results of these calculations are presented in Tables 1–5. Each table gives the results of a single capacity–rate ratio, and presents overall heat exchanger effectiveness, E, as a function of the counter-flow value of N_{tu} and the cross-flow (end section) value of N_{tu}. In other words the *total* value of N_{tu} for the heat exchanger is the *sum* of these two. These results can now be used for heat exchanger analysis and design in exactly the same manner as the $E - N_{tu}$ data presented in [1]. An interesting conclusion that can be drawn is that if the composite heat exchanger is treated as a counter-flow heat exchanger in series with a simple cross-flow heat exchanger, the result is very close to the exact solution over a wide range of operating conditions.

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